# Poster Summary: Towards Large-Scale Economic-Robust Spectrum Auctions

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## I. Introduction

Dynamic spectrum auction is an effective solution to manage spectrum across many small networks. Yet without proper economic design, spectrum auctions can be easily manipulated by bidders, suffering huge efficiency loss. Selfish bidders can manipulate their bids, individually or in groups, to game the system and obtain outcomes that favor their own interests and hurt others. Prior solutions have designed truthful auctions to tackle individual bidder cheating [8].

As the number of participants grows, collusion becomes a serious threat to auction revenue and efficiency. Extensive measurements [2], [3] have shown that in many past auctions including the FCC spectrum auctions, a small fraction ($<5\%$) of bidders have strategically formed one or multiple collusion groups and rig their bids to manipulate auction results, causing lower prices and unfair resource distribution. Because collusion is legally banned in commercial auctions, existing collusion groups were tacit and small in size, thus easier to form and hard (and expensive) to detect in large-scale auctions. Similar trends were observed in other practical deployments including P2P systems where each collusion group contains 2–4 players [5]. In this work, we show that in spectrum auctions, small-size collusion is even more effective than that in conventional auctions, because colluding bidders can exploit the bidder interference constraints to rig their bids. On the other hand, designed to addressing individual cheating, truthful auctions are highly vulnerable to collusion. These observations pose an important need for spectrum auctions to resist bidder collusion, particularly small-size collusion.

Prior solutions to tackle collusion in auctions, however, when applied to spectrum auctions, either cause severe interference or lose collusion-resistance [9]. This is because conventional designs [4], [6] do not consider any reuse and assume bidders have a homogeneous relationship: either all conflict with each other, or do not conflict at all. Whereas this relationship in spectrum auctions becomes heterogeneous due to the bidder interference constraints and the need for spatial reuse [8]. On the other hand, recent work on spectrum auctions focuses on suppressing some forms of collusion [7], but can be attacked easily by other simple forms. More importantly, this solution requires an exponential-complexity algorithm to ensure its resistance, thus cannot operate in large-scale dynamic spectrum auctions.

In this work, we propose DC$^2$, a new collusion-resistant and computationally-efficient spectrum auction. Using a randomization technique, DC$^2$ resists collusion by diminishing the gain of any colluding group unless it becomes large (and hence hard to form and easy to be detected). Such diminishing returns leave bidders little or no incentive to collude. Meanwhile, DC$^2$ enables spatial reuse to improve auction revenue and efficiency. DC$^2$'s novel contribution is to let the auctioneer secretely perform a 3-stage "Divide, Conquer, and Combine" procedure after receiving bids. By judiciously designing its procedure, DC$^2$ successfully integrates an efficient spectrum allocation algorithm (in "Divide") with a novel economic mechanism (in "Conquer"), enabling spatial reuse and effectively controlling collusion.

## II. Challenges

Consider the following spectrum auction scenario: An auctioneer runs an auction periodically. Each time it auctions off $K$ channels to some $n(n >> 1)$ bidders who submit bids privately. Each bidder requests one channel and treats the $K$ channels homogeneously. The auctioneer determines allocations and prices based on the interference constraints among the bidders. An important requirement is to enable spatial reuse so non-conflicting bidders can reuse the same channel. Such requirement makes dynamic spectrum auctions fundamentally different from conventional auctions, and imposes significant design challenges.

Collusion occurs in an auction when groups of bidders coordinate their bids to game the system, gaining unfair advantages and harming others. Multiple collusion groups might appear in a single auction, but each collusion group is rational. With no knowledge on other bidders' bids and behaviors (because the auctions are sealed-bid), a collusion group will only rig the bids if this can improve the group utility. The group utility refers to the sum of each member's utility, which is defined as its valuation minus its price paid for being a winner, otherwise 0.

**Small-Size Collusion is the Bottleneck.** Based on existing measurement studies and empirical analysis [2], [3] on past auctions including spectrum auctions, individual collusion groups are generally small yet highly effective. Furthermore, our own analysis shows that small-size collusion is even more effective in dynamic spectrum auctions because of the heterogeneous interference constraints. Varying one bidder's bid can create "***chain effects***" and affect auction results at many other bidders, opening up new vulnerabilities to collusion. Detailed examples are shown in [9].

**Why Existing Solutions Fail?** Existing solutions that tackle collusion can be categorized into two types: (1) ***Solution tackling some forms of collusion:*** [7] resists two specific forms of collusion in spectrum auctions (loser collusion and winner sublease). We show that it is highly vulnerable to a simple winner-loser collusion [9]. In addition, by relying on an exponential complexity algorithm, this solution is only applicable in small-scale auctions; (2) ***Solutions tackling all forms of collusion:*** posted price [4] is the only solution addressing all forms of collusion, yet it leads to unbounded loss in revenue. [4], [6] consider the notion of *soft collusion-resistance* developed for conventional auctions without any reusability. They either suffer severe interference or lose collusion-resistance because strategic bidders can utilize the interference constraints and rig their bids to change the allocation outcomes [9].

Given no practical solution for spectrum auctions that can resist any type of, especially small-size collusion in large-scale auctions, this motivates us to explore new designs.

## III. DC$^2$: DIVIDE, CONQUER, COMBINE

We introduce DC$^2$, a new auction design that resists *any* form of collusion while enabling spatial reuse. DC$^2$ applies the concept of soft collusion-resistance [4]– diminishing the gain of any collusion group by making the auction outcomes "insensitive" to bid changes of the group.

Consider $n$ bidders among which $\pi$ is one of the colluding groups and has $t$ bidders. In $\pi$'s view, if $\mathbb{B}$ is the bids of all $n$ bidders when $\pi$ does not cheat and $\mathbb{B}'$ is the bids when $\pi$ cheats, then $\mathbb{B}$ and $\mathbb{B}'$ differ by no more than $t$ bids. Being rational, $\pi$ will have little incentive to cheat if with a probability $p$ or higher, the auction procedure $f_{DC^2}$ returns the same auction price and result $\Gamma$ over $\mathbb{B}$ and $\mathbb{B}'$: $f_{DC^2}(\mathbb{B}) = f_{DC^2}(\mathbb{B}') = \Gamma$. The corresponding soft collusion-resistance is defined as the $(t, p)$-truthfulness [1], [4]:

***Definition** 1:* An auction achieves the $(t, p)$**-truthfulness** if with a probability of $p$ **or higher**, no collusion group of size $t$ or less can improve its group utility by rigging their bids.

### A. Design

While prior works fail when applied to spectrum auctions with spatial reuse [9], DC$^2$ overcome this challenge using the concept of "Divide and Conquer," integrating a spectrum allocation with a collusion-resistant design. In DC$^2$ the auctioneer secretely performs the following steps:

**Divide:** The auctioneer applies a spectrum allocation algorithm to divide the bidders independent of their bids into several non-overlapping *sub-markets* such that in each sub-market no bidders interfere with each other. Table 1 lists the procedure of forming $V$ sub-markets.

**Table 1. Forming Sub-Markets in DC$^2$**

| | |
|---|---|
| **STEP 1** | Apply a spectrum allocation algorithm to assign each bidder with one virtual channel, *independent of the bids*. |
| **STEP 2** | Group bidders with the same channel into a sub-market $\Phi_m$. |
| **STEP 3** | Return sub-markets: $\Phi_1, ..., \Phi_m, ..., \Phi_V$. |

**Conquer:** In each sub-market, the auctioneer applies a classical collusion-resistant mechanism tCP [4] for a "*virtual clearing*" to set a *virtual price* $\Gamma_m$ for each sub-market $\Phi_m$. Then it identifies $N(\Gamma_m)$ potential winners as those bidding no less than $\Gamma_m$, and treats them together as a *super bidder* representing the current sub-market. More importantly, to translate individual sub-market's collusion-resistance into the resistance in the entire auction, DC$^2$ judiciously design each super bidder $\Phi_m$' bid as an estimated revenue:

$$\hat{R}(\Gamma_m) = \Gamma_m \times g_c(N(\Gamma_m)), \quad (1)$$

where $g_c(\cdot)$ is a random rounding function that makes $\hat{R}(\Gamma_m)$ insensitive to $N(\Gamma_m)$ which could be affected by collusive bids. This procedure is the key to ensure collusion-resistance of the integration. Table 2 summarizes the actions.

**Table 2. Virtual Clearing in Sub-Market $\Phi_m$**

| | |
|---|---|
| **STEP 4** | 1) Apply tCP in $\Phi_m$ to set the virtual price $\Gamma_m$;<br>2) Compute an estimated revenue $\hat{R}(\Gamma_m)$. |
| **STEP 5** | 1) Identify the potential winners $W^{\Phi_m}$ as those with bids no less than $\Gamma_m$;<br>2) Represent $W^{\Phi_m}$ as a super bidder with bid $\hat{R}(\Gamma_m)$. |
| **STEP 6** | Return $\hat{R}(\Gamma_m)$, $W^{\Phi_m}$, and $\Gamma_m$. |

**Combine:** Given the available channels, the auctioneer selects winning sub-markets, in which the potential winners become final winners and each is charged by its sub-market's virtual price determined in "Conquer". We cannot recycle the losers of one winning sub-market to another, because it breaks the requirement of bid-independent sub-market formation that is essential to ensure collusion-resistance. We summarize the actions in Table 3.

**Table 3. Final Clearing in DC$^2$**

| | |
|---|---|
| **STEP 7** | Choose $\min(K, V)$ highest super bidders (sub-markets) as winners; assign one channel to each. |
| **STEP 8** | In each winning sub-market $\Phi_m$, its virtual winners $W^{\Phi_m}$ are real winners; each gets a channel and is charged by $\Gamma_m$. |

### B. Main Results

DC$^2$ faces two main issues in its design: First, the integration requires judicious design to ensure collusion-resistance, because now colluders can manipulate their bids to affect not only the auction result in each sub-market, but also the sub-markets they are assigned to; Second, a key component of DC$^2$ is to configure its auction procedure to maximize the auction revenue while guaranteeing the required level of collusion-resistance defined by the $(t, p)$-truthfulness. As following, we have two theorems as the main results on DC$^2$'s collusion-resistance, and its revenue bound by DC$^2$'s detailed configuration to maximize auction revenue for a given $(t, p)$. We refer the reader to [9] for the detailed proofs.

***Theorem** 1:* ***DC$^2$'s collusion resistance:*** *DC$^2$ achieves the $(t, p)$-truthfulness with $p = 1 + \log_{c_{min}}(1 - \lambda t/(l_{min} - t))$ where $l_{min}$ is the number of winners of the smallest sub-market using tCP for virtual clearing, $c_{min}$ and $\lambda$ are auction parameters. When $t/l_{min} << 1$, we have $p = 1 - O(t/l_{min})$.*

From the above theorem, we see that $l^{min}$ is critical to DC$^2$'s collusion-resistance. We particularly show in [9] that tCP is only applicable to all the sub-markets only when there are enough winners in each. Yet the sub-market configuration depends on the bidder interference constraints and the spectrum allocation algorithm. In many cases, there will be some small sub-markets that do not satisfy the $l_{tcp}$ condition. In this case, DC$^2$ applies tCP to only large enough sub-markets, and the posted price to others to achieve a better trade-off. Because the posted price mechanism achieves hard collusion-resistance, DC$^2$'s collusion-resistance $(t, p)$ only depends on the sub-markets that run tCP.

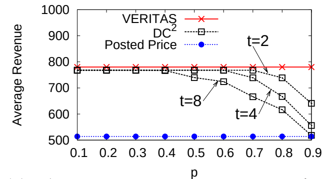
***Theorem 2: DC$^2$'s revenue bound:*** *While satisfying the $(t, p)$-truthfulness, DC$^2$ with $V$ sub-markets running tCP achieves a revenue no less than $R^{OPT}/(c^{max}\alpha^l_{tCP})$, where $R^{OPT}$ is the sum of the optimal revenue obtained by treating each of the $V$ sub-markets separately, and $c^{max}$, $\alpha^l_{tCP}$ are auction parameters required to achieve the $(t, p)$-truthfulness.*

To maximize revenue at given $(t, p)$ level, the main challenges facing DC$^2$ are due to the interdependency of sub-market configurations. This requires DC$^2$ to jointly configure sub-markets to achieve better global revenue, and also creates a dilemma in choosing balanced or imbalanced sub-market partitions. We design a statistical method for DC$^2$ to estimate the optimal configuration of $(l_{tCP}, \alpha^l_{tCP})$ in a bid-independent manner. The details of the configuration algorithm can be found in [9]. We also explore in [9] the impact of sub-market formation by integrating various spectrum allocation algorithms into DC$^2$. Our finding is that DC$^2$ favors the partition producing the large sub-markets. While existing allocation algorithms perform similarly, finding the optimal allocation algorithm is still an open research.

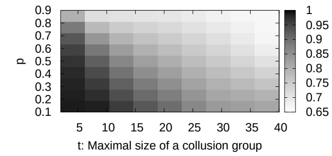
## IV. Evaluation

Using large-scale auction systems, we identify DC$^2$'s tradeoff between collusion-resistance and auction revenue by comparing three solutions: (1) **Posted Price** [4]: the only solution achieving hard resilience to collusion ($t = n, p = 1$). It picks a price randomly independent of bids; (2) **DC$^2$** with a soft $(t, p)$ collusion-resistance where $t$ is the maximal per-group size of all collusion groups; (3) **VERITAS** [8]: a truthful spectrum auction design that cannot address collusion. Focusing on large-scale auctions, we did not examine [7] because it only applies to small networks and resists some special forms of collusion. We did not compare soft collusion-resistant solutions in [4], [6] since their allocations are not conflict-free and simple extensions lose collusion-resistance [9].

Figure 1(a) compares the three solutions in terms of the average revenue, where the revenue of DC$^2$ depends on the required $(t, p)$. Compared to VERITAS, DC$^2$ sacrifices 5%, 14% and 21% revenue to achieve collusion-resistance with $t \leq 2, 4, 8$, and $p = 0.8$. This demonstrates DC$^2$'s effectiveness in resisting small-size collusion groups. Compared to posted price, DC$^2$ improves the revenue by as much as 50% even with $p = 0.8$. As the required collusion-resistance




(a) Average revenue vs. $p$ when the maximal collusion group size $\leq 2, 4, 8$.




(b) DC$^2$'s normalized revenue indicated by the darkness as a function of $(t, p)$.

Fig. 1. Tradeoff between resilience and revenue. (a) compares various designs; (b) shows the ratio of DC$^2$'s revenue over VERITAS at various $(t, p)$.

$(t, p)$ gets stronger, the number of sub-markets running tCP reduces and the revenue decreases. Eventually, DC$^2$ falls back to posted price. Overall, DC$^2$ introduces collusion-resistance to spectrum auctions at very little overhead, and offers an important flexibility of configuring the auction to exploit the tradeoff between revenue and collusion-resistance. Figure 1(b) further examines this tradeoff by showing the normalized revenue of DC$^2$ over VERITAS for various $(t, p)$. As expected, DC$^2$'s revenue decreases as $(t, p)$ increases. We see that for $t \leq 5$, the revenue degradation is significantly lower. This again verifies that DC$^2$ is effective over small-size collusion group, the dominant type in practical auctions.

## V. Conclusion and Future Work

To our best knowledge, DC$^2$ is the first to address any form of collusion in spectrum auctions and enable spatial reuse. DC$^2$ can be extended in several directions. (1) We can consider a different form of soft collusion-resistance by bounding the average collusion gain, which requires a mechanism design different from tCP. (2) Collusion will have more profound impact if bidders can request multiple channels. Addressing collusion in this context requires a stronger rule. (3) DC$^2$ can use any spectrum allocation algorithm. While the most well-known algorithms perform similarly, it is desirable to find the best allocation algorithm in DC$^2$ that maximizes the revenue.

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