TRUST: A General Framework for Truthful Double Spectrum Auctions (Extended)

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Abstract— **We design truthful double spectrum auctions where multiple parties can trade spectrum based on their individual needs. Open, market-based spectrum trading motivates existing spectrum owners (as sellers) to lease their selected idle spectrum to new spectrum users, and provides new users (as buyers) the spectrum they desperately need. The most significant challenge is how to make the auction economic-robust (truthful in particular) while enabling spectrum reuse to improve spectrum utilization. Unfortunately, existing designs either do not consider spectrum reuse or become untruthful when applied to double spectrum auctions. We address this challenge by proposing TRUST, a general framework for truthful double spectrum auctions. TRUST takes as input any reusability-driven spectrum allocation algorithm, and applies a novel winner determination and pricing mechanism to achieve truthfulness and other economic properties while significantly improving spectrum utilization. To our best knowledge, TRUST is the first solution for truthful double spectrum auctions that enable spectrum reuse. Our results show that economic factors introduce a tradeoff between spectrum efficiency and economic robustness. TRUST makes an important contribution on enabling spectrum reuse to minimize such tradeoff.**

I. Introduction

Due to their perceived fairness and allocation efficiency [13], auctions are among the best-known market-based mechanisms to distribute spectrum. In a well-designed auction, everyone has an equal opportunity to win and the spectrum is sold to bidders who value it the most. In the past decade, the FCC (Federal Communications Commission) and its counterparts across the world have been using *single-sided* auctions to assign spectrum to wireless service providers in terms of *predetermined* national/regional long-term leases.

In this paper, we show that auctions can be designed to dynamically redistribute spectrum across multiple parties to meet their own demands. As shown in Figure 1, the auctioneer runs *double* spectrum auctions to enable multiple sellers and buyers to trade spectrum dynamically. In this way, existing spectrum owners (as sellers) can obtain financial gains by leasing their selected idle spectrum to new spectrum users; new users (as buyers) can access the spectrum they desperately need and in the format they truly desire. By multiplexing spectrum supply and demand in time and space, dynamic auctions can significantly improve spectrum utilization.

In addition to enabling dynamic trading, our proposed spectrum auctions recognize that spectrum is reusable among bidders and exploit such reusability to improve auction efficiency. Unlike conventional FCC-style auctions that target only national service providers, our auctions allow buyers to be

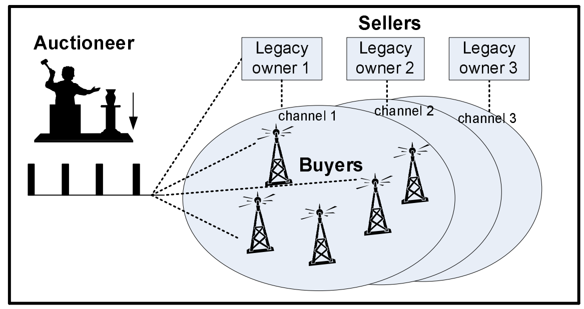


Fig. 1. Multi-party spectrum trading based on double auctions. The auctioneer performs an auction among both sellers and buyers. Sellers provide idle spectrum pieces dynamically with regional coverage, while buyers request spectrum channels in local areas based on their demands. Each channel contributed by a seller can be reused by multiple non-conflicting buyers.

small wireless networks, individual infrastructure networks or home networks. These small buyers seek local spectrum usage and thus can reuse spectrum in space. To distribute spectrum efficiently, one must exploit such spatial reusability.

The reusability, on the other hand, makes spectrum fundamentally different from conventional goods (*e.g.* paintings and bonds), and introduces significant difficulties in designing economic-robust auctions. In particular, truthfulness (or strategy-proofness) is one of the most critical properties required to implement an auction. Auctions without this property are extremely vulnerable to market manipulation and produce very poor outcomes, shown by both economic theory and concrete examples [12]. Unfortunately, conventional truthful double auction designs [1], [14] do not consider reusability, and prior work on truthful spectrum auctions [25] only addresses single-sided buyer-only auctions. When applied to double spectrum auctions, these designs and their extensions either cannot enable spectrum reuse or lose the truthfulness.

In this paper, we propose a framework for TRuthful doUble Spectrum aucTions (TRUST), achieving truthfulness and enabling spectrum reuse in double auctions. TRUST takes as input any reusability-driven spectrum allocation algorithm, and applies a novel winner determination and pricing mechanism to select winning sellers and buyers. To our best knowledge, TRUST is the first framework to address truthful double spectrum auctions with spectrum reuse.

TRUST makes the following key contributions.

- TRUST provides an efficient and trust-worthy environment for spectrum sellers and buyers to trade spectrum.

Instead of performing one-to-one trading, a seller can distribute its spectrum to many "smaller" buyers based on their demands.
- Tightly integrating spectrum allocation and pricing components, TRUST not only achieves truthfulness but also significantly improves spectrum utilization.
- TRUST can use any spectrum allocation algorithm, allowing the auctioneer to implement auctions based on customized performance/complexity requirements.
- In addition to truthfulness, TRUST also guarantees two economic properties, *individual rationality* and *ex-post budget balance*. Together, these three properties ensure that auctions are economic-robust and encourage bidder participation.

Through theoretical analysis and experimental results, we examine the performance of TRUST and study the impact of economics on spectrum distribution. Our results reveal the following findings:

- TRUST tradeoffs spectrum efficiency to achieve economic robustness. This tradeoff is necessary to resist selfish bidding behaviors that can lead to uncontrollable damage to auction efficiency. TRUST makes an important contribution on minimizing such tradeoff.
- While TRUST can operate on any spectrum allocation algorithm, economic factors can have a heavy impact on the choice of allocation algorithm. An allocation algorithm that is optimal in non-auction settings could perform worse than a randomized one in double spectrum auctions.
- Bid patterns can also affect the auction results significantly. In particular, the effect of economics diminishes quickly as the variance of buyer bids decreases.

II. Preliminaries

In this section, we provide an overview of the double spectrum auction problem, and discuss the critical economic properties required to implement the auction.

A. Problem Model

We consider a single-round double spectrum auction with one auctioneer, M sellers, and N buyers. We consider the most common spectrum trading scenario: large broadcasting providers selectively lease out their regional ownerships of spectrum channels to small wireless nodes like WiFi access points. In this case, sellers and buyers differ significantly in their spatial coverage (Figure 1). Each channel contributed by sellers can potentially be reused by multiple non-conflicting buyers. Exploiting such *reusability* is required to distribute spectrum efficiently. We focus on spatial reuse and thus assume sellers and buyers have the same time terms.

We assume that each seller contributes one distinct channel and each buyer only requests one channel. The channels are homogenous to buyers so that their requests are not channel-specific. The auction is sealed-bid and private. Bidders submit their bids privately to the auctioneer without any knowledge of others, and do not collude.

The problem of double spectrum auctions is defined as follows: "Given bids from sellers and buyers, how to choose winning sellers and their channels, assign these channels to selected winning buyers in a conflict-free manner, and determine the prices to pay/charge auction winners?" As discussed earlier, the goal of our auctions is to improve spectrum utilization and eliminate the artificial spectrum scarcity. To achieve this goal, the auctioneer focuses on enabling spectrum reuse and maximizing the number of trades. Instead of maximizing its revenue, the auctioneer maintains a non-negative budget and gains additional financial returns from service charges. In addition, the auctioneer must design auctions to achieve economic properties required to resist market manipulation. These economic properties makes the problem of double spectrum auctions significantly different from the conventional spectrum allocation problem.

B. Required Economic Properties

Truthfulness, individual rationality and budget balance are the three critical properties required to design economic-robust double auctions [2], [12]. To define them formally, we first introduce the following notations: bid, true valuation, clearing price, and bidder utility:

For a seller m, B^s_m is its bid, the minimum payment required to sell a channel; V^s_m is its true valuation of the channel; P^s_m is the actual payment received if it wins the auction; and its utility is $U^s_m = P^s_m - V^s_m$ if it wins the auction, and 0 otherwise. For a buyer n, B^b_n is its bid, the maximum price it is willing to pay for a channel; V^b_n is its true valuation of a channel; P^b_n is the price it pays if it wins the auction, and its utility is $U^b_n = V^b_n - P^b_n$ if it wins, and 0 otherwise. Note that the auction will have multiple winning sellers and buyers.

We now define the three economic properties:

(1) Truthfulness. A double auction is *truthful* if no matter how other players bid, **no** seller m or buyer n can improve its own utility by bidding untruthfully ($B^s_m \neq V^s_m$ or $B^b_n \neq V^b_n$).

Truthfulness is essential to resist market manipulation and ensure auction fairness and efficiency. In untruthful auctions, selfish bidders can manipulate their bids to game the system and obtain outcomes that favor themselves but hurt others. In truthful auctions, the dominate strategy for bidders is to bid truthfully, thereby eliminating the fear of market manipulation and the overhead of strategizing over others. With the true valuations, the auctioneer can allocate spectrum efficiently to buyers who value it the most.

(2) Individual Rationality. A double auction is *individual rational* if no winning seller is paid less than its bid and no winning buyer pays more than its bid:

$$P^s_m \geq B^s_m, \quad P^b_n \leq B^b_n \;\; \forall \text{ seller } m, \text{ buyer } n \tag{1}$$

This property guarantees non-negative utilities for bidders who bid truthfully, providing them incentives to participate.

(3) Ex-post Budget Balance. A double auction is *ex-post budget balanced* if the auctioneer's profit $\Phi \geq 0$. The profit is defined as the difference between the revenue collected from buyers and the expense paid to sellers:

$$\Phi = \sum_{n=1}^{N} P_n^b - \sum_{m=1}^{M} P_m^s \geq 0. \quad (2)$$

This property ensures that the auctioneer has incentives to set up the auction. Note that in practice the auctioneer can charge a transaction fee to (winning) bidders. For simplicity, we do not include this charge in the profit computation.

III. Challenges of Double Spectrum Auction Design

To enable efficient spectrum trading, the auction design must exploit spectrum reusability to improve spectrum utilization and achieve the three critical economic properties. The reusability, however, introduces significant difficulties in achieving economic robustness. As highlighted by the following table, conventional truthful double auction designs (VCG [1] and McAfee [14]) do not consider reusability. Prior work on truthful spectrum auctions (VERITAS [25]) only addresses single-sided buyer-only auctions, and loses the truthfulness when directly extended to double auctions.

Existing double auction designs	**Spectrum Reuse**	**Truthfulness**	**Ex-post Budget Balance**	**Individual Rationality**
VCG	X	√	X	√
McAfee	X	√	√	√
VERITAS extension	√	X	√	√
TRUST	√	√	√	√

In the following, we briefly summarize these existing designs and the lessons learned from applying them and their extensions to double spectrum auctions. Our proposed design, TRUST, is motivated by these observations, but achieves spectrum reuse and all three economic properties.

A. McAfee Double Auctions

Most existing double auctions use McAfee's design [14], which achieves the three economic properties but does not consider reusability. This design matches buyers to sellers one by one to make the auction profitable, but sacrifices the least profitable trade to achieve truthfulness. We can summarize this design by the following procedure:

(1) Sort bids in non-decreasing (for sellers) and non-increasing (for buyers) orders:

$$B_1^s \leq B_2^s \leq ... \leq B_M^s$$
$$B_1^b \geq B_2^b \geq ... \geq B_N^b$$

(2) Find $k = \operatorname{argmax} B_k^s \leq B_k^b$, the index of the least profitable transaction. The first $(k-1)$ sellers and the first $(k-1)$ buyers are the auction winners.

(3) Charge all the winning buyers equally by the bid of the kth ranked buyer B_k^b. Pay all the winning sellers equally with the bid of the kth ranked seller B_k^s.

To extend McAfee's design to consider spectrum reusability, one should map multiple non-conflicting buyers to each seller. This is also the motivation behind TRUST. However, the challenge is how to select such mapping and the pricing mechanism such that the design can still maintain the economic properties. As we will show, simple pricing models make the auction untruthful. On the other hand, TRUST follows the methodology of McAfee's design, but judiciously redesigns the mapping and pricing mechanisms to enable spectrum reuse and achieve the economic properties.

B. VCG Double Auctions

The VCG double auction model [1] is the same as McAfee's design except the choice of winners and their prices. Instead of choosing the top $(k-1)$ seller/buyer pairs, it chooses the top k pairs without sacrificing any trade. It charges each of the k winning buyers by $P^b = \max(B_{k+1}^b, B_k^s)$, and pays each of the k winning sellers by $P^s = \min(B_{k+1}^s, B_k^b)$. Unfortunately, $P^b \leq P^s$ and the auctioneer's profit can become negative ($\Phi < 0$), violating the property of budget balance.

C. Extension of Single-sided Truthful Spectrum Auction

We recently proposed VERITAS [25], a single-sided spectrum auction that enables spectrum reuse and achieves truthfulness. Being a single-sided auction, VERITAS only considers buyers. A direct extension to double auctions is to combine the VERITAS auction for the sellers, and one truthful auction for the buyers. Each single-sided auction requires the knowledge of k, the number of channels involved. For each k, the auctioneer performs a VERITAS auction among buyers assuming k channels are available for auctioning, and performs a truthful auction among sellers assuming the auction needs to collect k channels. Among all possible values of k, the auctioneer chooses the optimal k_{opt} that maximizes spectrum utilization with a non-negative auction profit. Based on k_{opt} and the corresponding auction result, the auctioneer chooses the auction winners and their prices.

This extension, however, is not truthful. Because seller and buyer auctions are disjoint, a buyer can strategically plan its bid to reduce its clearing price while winning the auction. Let us illustrate it with an example. Assume there are four sellers who bid truthfully with 1, 2, 3, and 4 respectively. Assume there are four buyers a, b, c, and d with the conflict graph of a line topology (Figure 2). Figure 3 shows the results when all buyers bid truthfully with $6, 5, 4$ and 1 respectively. V^b, B^b, A^b, U^b denote the valuation, bid, allocation, and utility of the buyer. In this case, the auctioneer can only purchase one channel from the sellers and sell it to the buyers while maintaining a non-negative profit. Hence, buyer d loses and has a zero utility. On the other hand, if d lies by raising his bid to 3, the auctioneer will enable two channels transacted because it produces a positive profit of 1. As a result, c's utility drops, and buyer d wins with the utility of one, violating the

Fig. 2. The conflict graph of buyer a, b, c, and d.

# of channels to trade	Revenue	Payment
1	**5+1=6**	**2**
2	4+1=5	2*3=6
3	0	3*4=12
Result	Trade 1 channel	

Winning seller: Seller 1

	a	b	c	d
V^b	6	5	4	1
B^b	6	5	4	1
A^b	1	0	1	0
U^b	1	0	3	0

(a) All bidders bid truthfully

# of channels to trade	Revenue	Payment
1	5+3=8	2
2	**4+3=7**	**2*3=6**
3	0	3*4=12
Result	Trade 2 channels	

Winning sellers: Seller 1 and 2

	a	b	c	d
V^b	6	5	4	*1*
B^b	6	5	4	*3*
A^b	1	1	1	*1*
U^b	6	1	1	*1*

(b) Buyer *d* lies by raising bid to 3, all other bidders bid truthfully

Fig. 3. Comparison of the allocation results when buyer d (a) bids truthfully or (b) raises his bid to 3. For (a), the auctioneer buys 1 channel, and thus d is denied with the utility of zero. For (b), the auctioneer buys 2 channels while maintaining ex-post budget balance, and d obtains a channel and also a positive utility.

property of truthfulness. So d rigs its bid and reduces other's utility. This lesson shows that the seller and buyer auctions, particularly the pricing, need be jointly performed together.

D. Achieving Economic Robustness and Efficiency

From the above, we see an immediate need for a new double spectrum auction design that can enable spectrum reuse to maximize spectrum utilization while being economic-robust. On the other hand, the *Impossibility Theorem* [16] shows that no double auctions can simultaneously achieve all three economic properties while maximizing auction efficiency. Because the three economic properties are necessary to implement the auction, the design should focus on satisfying them first while approximately maximizing efficiency. This is also the general approach used by existing double auction designs [1], [2], [7], [10].

IV. TRUST: Design Rationale

TRUST follows the methodology of McAfee's design, but breaks the barrier between spectrum reuse and economic-robustness. It tightly integrates a reusability-driven spectrum allocation algorithm with a pricing mechanism. The essence of our design is to map a group of buyers into each seller and to choose the mapping and the pricing judiciously. We discuss the design rationales in this section, and present its detailed design and the proofs of its economic properties in Section V.

(1) Bid-independent Buyer Grouping

The first question is how to group multiple non-conflicting buyers together so that they can be assigned with the same channel. This maps to the spectrum allocation process. Like VERITAS [25], the allocation can depend on the bids. However, a bid-dependent allocation introduces a critical vulnerability to bid manipulation where selected buyers can strategically plan their bids to change the groups formed, the number of channels transacted, and hence their utilities. Therefore, in TRUST, we choose to form buyer groups based on their interference conditions but independent of their bids.

(2) Uniform Pricing within Each Buyer Group

After forming buyer groups, we can treat each group as a *super buyer* and reduce the problem settings to those in McAfee's design. The immediate question is how to determine the bid of each super buyer, and once a super buyer is selected, how to charge the buyers inside the corresponding group. To charge buyers in a winning group, a straightforward approach is to use discriminatory pricing, such as charging each buyer proportionally to its bid. However, this could make the auction untruthful because selective buyers in a winning group can manipulate their bids to lower their shares in the group charge while still winning the auction. Now by lowering their clearing prices, they improve their utilities, violating the truthfulness requirement. Further, this conclusion holds no matter how the group bid is computed.

While the design of discriminatory pricing within each group is an interesting open problem, we propose to use uniform pricing in each group. Uniform pricing is in fact fair because buyers in a winning group obtain the same channel, thus should be charged equally. Under uniform pricing, we show that the following result holds:

Theorem 1: *Under per-group uniform pricing, to make the auction individual-rational and truthful, the group bid needs to be no more than the product of the lowest buyer bid in the group and the number of buyers in the group.*

Proof: First, individual rationality requires that each buyer must not be charged higher than its bid. Under uniform pricing, buyers in the same group are charged equally. Therefore, the total price charged to a winning group i, p_i, must not be higher than the lowest bid b_i^{min} in this group times the group size n_i, $p_i \leq b_i^{min} \cdot n_i$. Second, as shown in McAfee's design, to maintain truthfulness, the price charged to a winning group i is the bid of another group k that is no higher than its own group bid π_i: $p_i = \pi_k \leq \pi_i$. Combine these two conditions together and take the extreme scenario $p_i = \pi_i$, we have $\pi_i \leq b_i^{min} \cdot n_i$. ■

We use an example to illustrate the above theorem. Consider 3 sellers with bid 4, 5 and 6, and 3 buyer groups $G_1 = \{a, b\}, B_a = 2, B_b = 8$, $G_2 = \{c, d, e\}, B_c = 1, B_d = 2, B_e = 3$, and $G_3 = \{f, g\}, B_f = 2, B_g = 3$. If we use the total sum of bids as the group bid, they are 10, 6, and 5 respectively. G_1 is the winning group, and is charged by 6, so bidder a and b will be charged by $6/2 = 3$ which is higher than a's bid. But if $\pi_i = b_i^{min} \cdot n_i$, then G_1 wins, and a and b are charged with 2.

V. TRUST: Design Details and Proofs

Following the guidelines in Section IV, we now describe TRUST in detail and prove analytically that it achieves the three economic properties. TRUST performs the auction in three steps. TRUST first takes as input any spectrum allocation algorithm to form buyer groups. Then by comparing the seller bids with the buyer group bids, TRUST chooses the top $(k-1)$ seller/buyer group pairs as the auction winners. Finally, TRUST uses the kth seller bid and the kth buyer group bid to charge the winning sellers and buyers.

In essence, TRUST enables spectrum reuse by using a reusability-driven spectrum allocation algorithm to form buyer groups. It achieves the economic properties via the bid-independent group formation and a reusability-aware pricing mechanism. In addition, the flexible choice of the allocation algorithm also enables the auctioneer to customize the auction design to its own performance/complexity needs.

A. Detailed Procedures

TRUST consists of the following three steps:

Step I: Buyer Group Formation
TRUST performs a spectrum allocation assuming all the sellers' channels are available to the buyers. Buyers that are assigned to the same channel are organized into the same group. The group formation is performed *privately* by the auctioneer before the actual auction and kept confidential to the bidders. The group formation can cope with various interference models by using different spectrum allocation algorithms. If the buyer interference condition is modeled by a conflict graph, the group formation is equivalent to finding the independent sets of the conflict graph [17], [21]. If the condition is modeled by the physical Signal to Interference and Noise Ratio (SINR) [18], TRUST finds multiple sets of buyers who can transmit simultaneously and maintain an adequate received SINR [3]. We note that because channels are homogeneous, TRUST performs this allocation only to form buyer groups, not to assign specific channels to buyers.

Step II: Winner Determination
Let $G_1, G_2, ..., G_L$ represent the L groups formed in step I. For any group G_l with $n_l = |G_l|$ buyers, the group bid π_l is:

$$\pi_l = \min\{B^b_n | n \in G_l\} \cdot n_l. \tag{3}$$

TRUST sorts the seller bids in non-decreasing order and the buyer group bids in non-increasing order.

$$\mathbb{B}' : B^s_1 \leq B^s_2 \leq ... \leq B^s_M$$
$$\mathbb{B}'' : \pi_1 \geq \pi_2 \geq ... \geq \pi_L.$$

Define k as the last profitable trade:

$$k = \underset{l \leq \min\{L,M\}}{\operatorname{argmax}} \; \pi_l \geq B^s_l. \tag{4}$$

Then the auction winners are the first $(k-1)$ sellers represented by TOP($\mathbb{B}', k-1$), and the first $(k-1)$ buyer groups represented by TOP($\mathbb{B}'', k-1$). The detailed procedure is shown in Algorithm 1.

Algorithm 1 TRUST-DetermineWinner($groups$,B^b,B^s)

for $l = 1$ to L **do**
  $\pi_l = \min\{B^b_n | n \in G_l\} \cdot n_l$
end for
$\mathbb{B}'$ = sorted B^s in non-decreasing order
$\mathbb{B}''$ = sorted $\{\pi_i | 1 \leq i \leq L\}$ in non-increasing order
$k = \operatorname{argmax}_{l \leq \min\{L,M\}} \pi_l \geq B^s_l$
$winners$ = TOP($\mathbb{B}', k-1$) ∪ TOP($\mathbb{B}'', k-1$)
Return $winners$

Step III: Pricing
To maintain truthfulness, TRUST pays each winning seller m by the kth seller's bid B^s_k. TRUST charges each winning buyer group l by the kth buyer group's bid π_k. This group price is evenly shared among all the buyers in the group l:

$$P^b_n = \pi_k / n_l, \;\; \forall n \in G_l. \tag{5}$$

No charges or payments are made to losing buyers and sellers. The auctioneer's profit Φ is:

$$\Phi = (k-1) \cdot (\pi_k - B^s_k) \tag{6}$$

B. An Illustrative Example

In Figure 4 we consider an auction with 7 buyers (A-G) and 4 sellers. The interference conditions among the buyers are modeled by a conflict graph. Two buyers share an edge in the graph if they conflict with each other and cannot reuse the same channel. To illustrate the impact of allocation algorithms, we compare the auction results when TRUST uses two allocation algorithms, OPT and RAND. OPT refers to the optimal algorithm in the non-auction setting. It minimizes the number of channels required to provide each buyer a channel. RAND refers to a randomly produced allocation result.

As shown in Figure 4, TRUST with OPT produces 3 groups with group bids 7, 6 and 1.5. When compared with the sellers' bids $\{1, 2, 3, 4\}$, only $k = 2$ groups are higher than the corresponding sellers. Therefore, group 1 (*i.e.* buyer F) is the auction winner and only one channel is traded. On the other hand, TRUST with RAND produces 4 groups with group bids 8, 5, 4, and 0.5, leading to two winning groups $\{A, F\}$+$\{B, E\}$ and a spectrum utilization of 4. While in this example OPT underperforms RAND, there exist bid patterns for the same topology where OPT outperforms RAND. This somewhat surprising observation demonstrates the impact of economics on spectrum distribution. In Section VI we examine this problem again in detail using large-scale simulations.

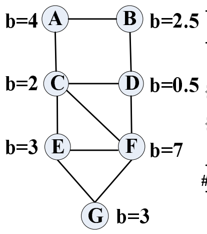
C. Proof of Auction Properties

We prove that TRUST satisfies the properties of ex-post budget balance, individual rationality, and truthfulness.

1) Proof of Ex-post Budget Balance:

Theorem 2: TRUST is ex-post budget balanced, *i.e.* $\Phi \geq 0$.

Proof: Because k is the largest index that satisfies $\pi_k \geq B^s_k$, from (6) it is straightforward to show that $\Phi = (k-1) \cdot (\pi_k - B^s_k) \geq 0$. ■



OPT		RAND		SELLER BIDS
Group	Group bid	Group	Group bid	
{F}	7	{A, F}	4*2=8	1
{B, C, G}	2*3=6	{B, E}	2.5*2=5	2
{A, D, E}	0.5*3=1.5	{C, G}	2*2=4	3
		{D}	0.5	4
#CH Traded	1	#CH Traded	2	
Spectrum utilization	1	Spectrum utilization	4	

Fig. 4. Comparing the auction results of integrating the optimal allocation and a random allocation.

2) Proof of Individual Rationality:

Theorem 3: TRUST is individual rational.

Proof: By the definition of individual rationality (Equation 1), we need to show that no winning seller will be paid less than its bid, and no winning buyer will be charged more than its bid.

First, because TRUST sorts seller's bids in a non-decreasing order and pays each winning seller m with the kth seller's bid, the payment to m is P^s_m= $B^s_k \geq B^s_m$. Second, for each winning buyer n in group G_l, G_l's group bid π_l must be no less than π_k since groups bids are sorted in non-increasing order. Then the price charged to n is $P^b_n = (\pi_k/n_l) \leq (\pi_l/n_l)$. By group bid's definition in Equation 3, we have $P^b_n \leq (\pi_l/n_l) \leq B^b_n$. ■

3) Proof of Truthfulness: To prove TRUST's truthfulness, we need to show that for any buyer n or seller m, it cannot improve its utility by bidding other than its true valuation. For this, we first show that its winner determination is monotonic for both sellers and buyers and the pricing is bid-independent. Using these two claims, we then prove the truthfulness.

(1) Monotonic winner determination

The following two lemmas summarize the monotonicity of TRUST's winner determination. Their proofs are in the Appendix.

Lemma 1: Given $\{B^b_1, ..., B^b_{n-1}, B^b_{n+1} ..., B^b_N\}$ and $\{B^s_m\}^M_{m=1}$, if buyer n wins the auction by bidding B^b_n, then buyer n also wins by bidding $B'_n > B^b_n$.

Lemma 2: Given $\{B^s_1, ..., B^s_{m-1}, B^s_{m+1} ..., B^s_M\}$ and $\{B^b_n\}^N_{n=1}$, if seller m wins by bidding B^s_m, then m also wins by bidding $B'_m < B^s_m$.

(2) Bid-independent pricing

We show that the pricing is bid-independent for both winning buyers and sellers. The proofs can be found in the Appendix.

Lemma 3: Given $\{B^b_1, ..., B^b_{n-1}, B^b_{n+1} ..., B^b_N\}$ and $\{B^s_m\}^M_{m=1}$, if buyer n wins the auction by bidding B^b_n and B'_n, the price P^b_n charged to n is the same for both.

Lemma 4: Given $\{B^s_1, ..., B^s_{m-1}, B^s_{m+1} ..., B^s_M\}$ and $\{B^b_n\}^N_{n=1}$, if seller m wins the auction by bidding B^s_m and B'_m, then the payment P^s_m to m is the same for both.

(3) TRUST's truthfulness

Using the above claims, we now prove the main results on TRUST's truthfulness.

Case	1	2	3	4
The bidder lies	X	X	$\surd$	$\surd$
The bidder bids truthfully	X	$\surd$	X	$\surd$

TABLE I

FOUR POSSIBLE AUCTION RESULTS WHEN BIDDING TRUTHFULLY AND UNTRUTHFULLY. X MEANS THE BIDDER LOSES AND $\surd$ MEANS HE WINS.

Theorem 4: *TRUST is truthful for buyers.*

Proof:

We need to show that any buyer n cannot obtain higher utility by bidding $B'_n \neq V^b_n$. Table I lists the four possible auction results for one buyer when it bids truthfully and untruthfully. We now examine these cases one by one.

- CASE 1: For both bids, buyer n is denied and charged with zero, leading to the same utility of zero.
- CASE 2: This happens only if $B'_n < V^b_n$ (Lemma 1). Theorem 3 ensures a non-negative utility when n bids truthfully and wins the auction. Thus, its utility is no less than than that when it bids untruthfully (zero utility). Our claim holds.
- CASE 3: This happens only if $B'_n > V^b_n$ (Lemma 1). Let π_l and $\pi_l{}'$ represent the group bid of n's group when n bids truthfully and untruthfully. Because n changes the auction results by bidding higher than V^b_n, n must be the lowest bidder in its group when it bids truthfully, *i.e.* $\pi_l = V^b_n \cdot n_l$. Next, because n loses by bidding V^b_n and wins by bidding B'_n, it is easy to show that the price charged to its group when it wins, p, must satisfy the following condition: $\pi'_l \geq p \geq \pi_l$. Therefore, the utility when n bids B'_n is $V^b_n - (p/n_l) \leq 0$, which is no more than when n bids truthfully (0). Our claim holds.
- CASE 4: According to Lemma 3, n is charged by the same price in both cases, leading to the same utility. Our claim holds.

From the above, we show that no buyer can improve its utility by bidding untruthfully, which completes our proof. ■

Theorem 5: *TRUST is truthful for sellers.*

Proof: Similarly, we need to show that any seller m cannot obtain higher utility by bidding $B'_m \neq V^s_m$. Again, we examine the four cases listed in Table I.

- CASE 1: The same as the buyer case.
- CASE 2: This happens when $B'_m > V^s_m$(Lemma 2). Because its utility is non-negative when m bids truthfully and wins (Theorem 3), our claim holds.
- CASE 3: This happens when $B'_m < V^s_m$ (Lemma 2). First, let p be the payment to the auction winners when m bids truthfully. Because m loses in this case, $p \leq V^s_m$. Second, let p' be the payment to the winners (including m) when m bids B'_m. It is easy to show that because m lowers its bid and wins, $p' \leq p$. Combine the two, we have $p' \leq V^s_m$ and hence m's utility when bids untruthfully is $p' - V^s_m \leq 0$. Our claim holds.

- CASE 4: According to Lemma 4, the payment for m does not change, leading to the same utility in both cases.

Having shown that no seller can improve its utility by bidding other than its true value, our proof completes. ■

VI. Experimental Results

In this section, we use network simulations to evaluate the performance of TRUST, and study the impact of economics on spectrum distribution.

A. Simulation Setup

We study the performance of TRUST under different settings. The key factors that affect TRUST's performance are the underlining spectrum allocation algorithms, the interference conditions among buyers, and the bid distributions. We assume that the buyer interference conditions are modeled by a conflict graph. All the results are averaged over 1000 rounds.

(1) Allocation Algorithm. We consider four well-known channel allocation algorithms to form buyer groups. Previously in Figure 4, we have compared the optimal allocation algorithm in non-auction settings (OPT) with a random allocation algorithm (RAND) using a small topology. In this section, we extend to large randomly generated topologies. Because of OPT's exponential complexity, we compare RAND with three polynomial-time solutions. We select these solutions because they are all guaranteed to be within some proven factors of the optimal solution (OPT) in non-auction settings.

- **Max-IS** [21]: It assigns channels by finding the maximum independent set of the conflict graph. To form each independent set, it recursively picks a buyer (or node) with the minimal maximum independent set in the induced subgraph in its neighborhood.
- **Greedy-U** [17]: To form a group, it recursively chooses a node with the minimal degree in the current conflict graph, eliminates the chosen node and its neighbors, and updates the degree values of the remaining nodes.
- **Greedy** [17]: it is the same as the Greedy-U except that it chooses the nodes based on its original degree value.
- **RAND**: it randomly picks a node to allocate a channel.

(2) Interference Condition. The auction performance depends on the interference condition among buyers. We model the interference condition using a conflict graph, and apply a distance-based criterion to determine whether two buyers conflict. In this case, the interference condition depends mainly on the network topology. We consider two types of topologies:

- **Random Topologies**: We randomly distribute a set of buyers in a given area, with an average conflict degree of 6.5.
- **Clustered Toplogies**: We randomly place some buyers in a given area and gradually add buyers in a small center area, creating a hotspot.

(3) Bid Distribution. By default, we assume that buyers' bids are randomly distributed over $(0, 1]$. To examine the impact of bid variance, we assume that each buyer's bid is defined by $v \cdot \alpha + (1 - v)$, where α is a random number uniformly distributed over $(0, 1]$. Each seller's bid is also uniformly distributed over $(0, f]$. f is the spectrum cost factor, and $f = 2$ by default.

The performance metrics are spectrum utilization (the total number of winning buyers), the number of channels traded and the per-channel spectrum utilization. We also consider auction efficiency (or social welfare), which is the bid-weighted sum of all winning buyers minus that of sellers [1]. We observe that spectrum utilization and auction efficiency reflect the same conclusions, thus omit the auction efficiency results.

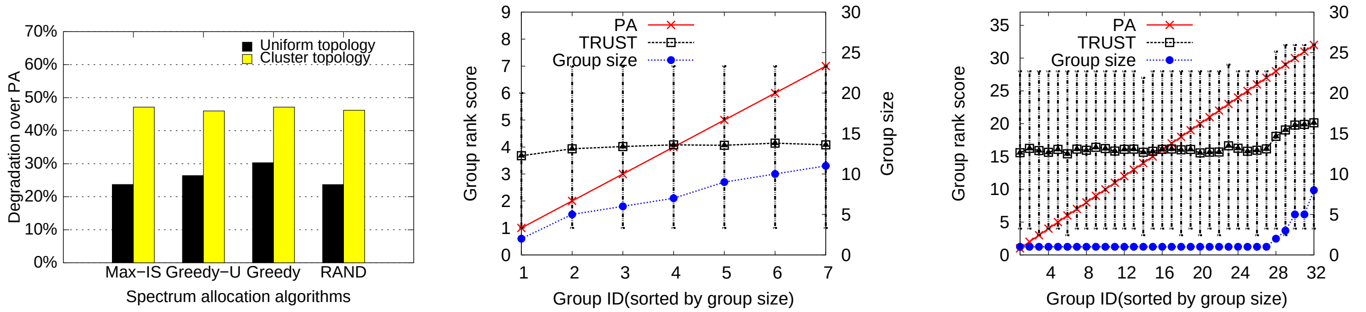
B. Economic Impact on Spectrum Distribution

We start from the impact of economics on spectrum distribution and compare TRUST to PA (Pure Allocation) in terms of spectrum utilization. Note that spectrum utilization depends on both the number of channels traded and the per-channel utilization. For a fair comparison, we implement PA using each allocation algorithm assuming the number of channels available is equal to the number of channels traded when the same algorithm operates in TRUST. In this way, the difference between TRUST and PA reflects the impact of economics on the per-channel utilization, or allocation efficiency. Note that by focusing solely on utilization maximization, PA is expected to outperform TRUST. We use this study to examine the impact of economics.

Figure 5(a) plots the degradation of TRUST over PA for both random and clustered topologies with 50 buyers and 10 sellers. The first observation is that TRUST suffers from notable degradations (20–50%) for all four allocation algorithms. Because TRUST and PA use the same allocation algorithm and hence produce the same set of buyer groups, the cause of degradation lies in the choice of winning buyer groups. Focusing solely on maximizing utilization, PA always chooses the groups with larger size, thus more spectrum reuse. On the other hand, to maintain economic-robustness, TRUST chooses the groups based on their group bids, the product of the group size and the minimum buyer bid in the group. Statistically, a larger group is likely to have a lower minimum bid, making it no longer preferable over smaller groups. We note that to achieve truthfulness, TRUST's group formation must be done independent of the buyer bids. Thus we cannot control the value of the minimum bid in each group.

Figure 5(b)-(c) demonstrate this trend in terms of the group rankings, where only the top $(k-1)$ (defined in Eq.(4)) groups get allocated. For a given topology and a set of buyer groups produced by Greedy-U, we plot the average group rankings with a confidence interval of 80% over 1000 random bid patterns. We see that PA ranks groups by their sizes, while TRUST ranks the groups almost equally. By selecting smaller groups, TRUST inevitably suffers from efficiency loss.

The second key observation is that TRUST's degradation increases to nearly 50% in clustered topologies. This is because as group size becomes much more diverse in clustered topologies, choosing smaller groups leads heavier efficiency loss. In the example of Figure 5(c), 26 of out of 32 groups



(a) Spectrum utilization (b) Group rankings in a random topology (c) Group rankings in a clustered topology

Fig. 5. The impact of economics in terms of the performance difference between TRUST and PA assuming 10 sellers and 50 buyers in random and clustered topologies. For the clustered topology, 20 out of 50 buyers are placed in the hotspot. (b) and (c) use Greedy-U. In (b) and (c), a group with a higher score is ranked higher by the auctioneer. We show the average rankings with a 80%([10%,90%]) confidence interval over 1000 bid patterns.

are of size 1 and the rest are of size 8 to 3. Furthermore, the following table summarizes the degradation of TRUST over PA as the number of buyers in the hotspot increases. We see that the degradation increases with the level of clustering.

♯ of cluster nodes	0	10	20	30	40
Degradation over PA	26.4%	45.4%	49.2%	50.5%	51.6%

Finally, we note that the degradation of TRUST over PA is not a disadvantage but a necessary tradeoff. In reality, spectrum is not free, so it cannot be "ideally" distributed without any economic consideration as in PA. TRUST aims at distributing spectrum efficiently by accounting economic factors and resisting market manipulations.

C. Choosing Allocation Algorithm in TRUST

The allocation algorithm is a key component of TRUST. The following table illustrates the average performance of the four allocation algorithms with TRUST. The per-channel spectrum utilization illustrates their performances without economic considerations. While Greedy and RAND are slightly inferior to Max-IS and Greedy-U in non-auction settings, they perform similarly in TRUST. This is because they have different number of channels traded. By producing more (and smaller) groups that lead to more chances to exceed sellers' bids, Greedy and RAND obtain more channels. This gain compensates their inefficiencies in the per-channel utilization.

	Spectrum utilization	**Per-channel utilization**	**♯ of channels transacted**
Max-IS	17.6880	7.4226	2.3830
Greedy-U	17.6910	7.3712	2.4000
Greedy	17.5550	6.8361	2.5680
RAND	17.5680	7.0526	2.4910

To effectively examine the impact of allocation algorithms, we perform a different experiment. Instead of going through different allocation algorithms, we limit the size of each buyer group (or the level of spectrum reuse) from 1 to the size of the largest group (34). We deploy 200 buyers and 10 sellers, producing at least 14 groups in a random topology. Therefore, the number of groups is always higher than the number of sellers. Results from random and clustered topologies reveal the same trend and hence only the results of random topology are shown. Note that TRUST with the size limit of 1 is McAfee's design.

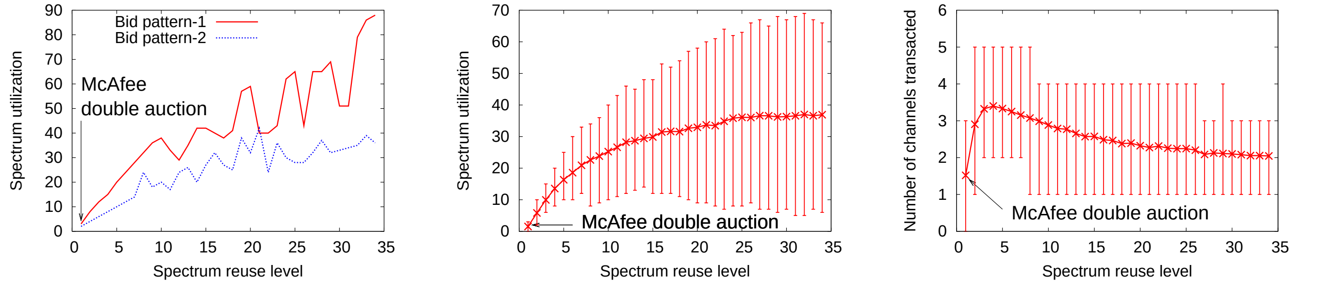
For a given topology and two specific bid patterns, Figure 6(a) shows the spectrum utilization as the level of spectrum reuse increases. We see that the spectrum utilization fluctuates randomly and maximizing the reuse level does not always perform well. We also examine the general trends by averaging the results over 1000 bid patterns. Figure 6(b) shows the average spectrum utilization with a 80% confidence interval. The average utilization initially increases quickly with the limit, indicating that an efficient allocation is important to TRUST. After the limit exceeds 20, the performance stabilizes. Yet, in Figure 6(c) the average number of channels transacted increases initially and then decreases. This is because, as the size of the buyer group increases, the number of groups decreases. Statistically, the number of groups with bids high enough to transact with the sellers decreases, reducing the set of channels traded. Thus, although the per-channel utilization increases as the limit grows, the spectrum utilization stabilizes.

Overall, the results show that the choice of the spectrum allocation algorithm in TRUST is indeed important, but can be very different from those in conventional settings. The economic factors randomize the choice of winning groups, hence the number of groups produced and their group sizes need to be judiciously designed. While TRUST can work with any allocation algorithm, finding an (approximately) optimal algorithm is an interesting problem worth exploring.

D. Impact of Bid Distribution

We study two key factors: the buyer bid variance and the ratio of seller to buyer bid f, or the spectrum cost factor. Regarding f, both the spectrum utilization and the number of channels transacted decrease as f increases. The results are intuitive and hence omitted in the interest of space. Next, we focus on examining the impact of the buyer bid variance.

The group bid of a buyer group depends heavily on the lowest bid, and thus the variance in buyer bids. To examine its impact, we produce buyer bids using $v \cdot \alpha + (1 - v)$, where



(a) Spectrum utilization for sample rounds (b) Average spectrum utilization (c) Average number of channels transacted

Fig. 6. The performance of TRUST at different limits of spectrum reuse with 200 buyers in a random topology. TRUST at spectrum reuse of 1 reduces to McAfee's design. Greedy-U is used for channel allocation. The results are averaged over 1000 bid distributions, with a 80%([10%,90%]) confidence interval.

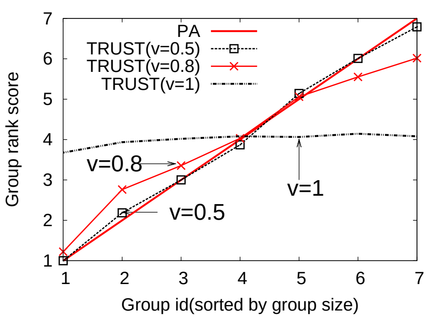


Fig. 7. The group rankings as the bid variance varies, with 10 sellers and 50 buyers, $f = 2$, and a random buyer topology.

α is a random number uniformly distributed over $(0, 1]$, and v defines the dynamic range of buyer bids. The following table summarizes the degradation of TRUST over PA for different v. We see that only when v exceeds 0.8, there is a notable difference between TRUST and PA.

v	0.5	0.6	0.7	0.8	0.9	1
Degradation over PA	0	$<$1%	1%	4%	10%	26%

This result can be explained by the group rankings in Figure 7. Intuitively, reducing the bid variance v also reduces the effect of the lowest bid on the group bid. Gradually, the group size becomes the dominate factor, and TRUST converges to that of PA. For the examples in Figure 7, when $v = 1$, the group rankings in TRUST are flat, and when v reduces to 0.5, the rankings converge to that of PA.

VII. Related Works

Auctions have been widely used to allocate spectrum [5]. Prior efforts include transmit power auctions [9], spectrum band auctions [4], [8], [19], [20], and spectrum pricing [6], [11], [15], [23]. However, they do not consider truthfulness. [25] is the first truthful spectrum auction design, but only addressed single-sided auctions. For double spectrum auctions, [24] proposes a hierarchical design based on McAfee's design without spectrum reuse. To the best of our knowledge, TRUST is the first truthful double auction design with spectrum reuse for multi-party spectrum trading. In addition, TRUST works with various spectrum allocation algorithms [17], [21], [22].

Truthfulness is a critical factor to attract participation [12]. Many truthful mechanisms have been developed in conventional double auctions, including single-unit [1], [7], [14] and multi-unit double auctions [2], [10]. The majority of these designs follow the idea of McAfee's mechanism [14], using the trade reduction to maintain truthfulness. TRUST differs significantly from these conventional designs in that it exploits the spectrum reusability to distribute spectrum efficiently.

VIII. Conclusion and Future Works

We propose TRUST, a truthful double spectrum auction framework to support dynamic multi-party spectrum trading. TRUST achieves truthfulness, individual rationality, and ex-post budget balance, the three key economic properties required for economic-robust auctions. More importantly, TRUST enables spectrum reuse to significantly improve spectrum utilization. From the design and evaluation of TRUST, we see that economic factors impact spectrum distribution heavily and introduce a tradeoff between spectrum efficiency and economic robustness. This tradeoff is necessary to resist selfish bidding behaviors that can lead to uncontrollable damage to auction efficiency. TRUST makes an important contribution on minimizing such tradeoff.

In this paper, we design TRUST based on a simple network scenario. To deploy TRUST in practice, several practical issues must also be addressed. First, TRUST assumes simple spectrum demand/request formats. Each buyer requests only one channel and yet in practice they can ask for multiple. Extending TRUST to address complex bidding formats is an interesting open research problem. Second, TRUST assumes that the auctioneer has complete information of buyer interference conditions. Yet in practice how to obtain this information reliably is still a challenge. Finally, there are additional economic properties that could be considered by the auction design to further increase its economic-robustness. For example, how to resist collusions and what is the tradeoff between efficiency and economic robustness are also interesting problems worth exploring.

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A. Proof of Lemma 1

Proof: Assume buyer n is in group G_l. Let G_l's group bid be π_l and π'_l when n bids B^b_n and B'_n respectively. Then we have $\pi'_l \geq \pi_l$ since $B'_n > B^b_n$. Therefore, G_l will rank higher in the sorted group bid list. Since n is allocated by bidding B^b_n, the group price must be determined by a group with the group bid lower than π_l. In other words, G_l with group bid π'_l will also be allocated. Our claim holds. ■

B. Proof of Lemma 2

Proof: Since sellers are ranked by their bids in the reverse order as buyers', we can similarly arrive at Lemma 2, which claims that if seller m wins by bidding B^s_m, then m also wins by bidding $B'_m < B^s_m$. ■

C. Proof of Lemma 3

Proof: Without the loss of generality, let $B^b_n < B'_n$. Assume buyer n is in group G_l, π_l and π'_l is G_l's group bid in the two cases respectively. Hence we have $\pi_l \leq \pi'_l$. Let $pos(\pi_l)$ and $pos(\pi'_l)$ denote G_l's positions in the sorted list of group bids, then $pos(\pi_l) \geq pos(\pi'_l)$, namely the change of G_l's position will not affect the groups ranked after $pos(\pi_l)$. This means that the group price does not change in both cases since it is determined by one of those groups. Moreover, n's bid does not affect G_l's members, therefore the price P^b_n for buyer n remains the same. ■

D. Proof of Lemma 4

Proof: Since seller m wins the auction by bidding B^s_m and B'_m, the payment is determined by a seller ranked after m, which does not change in both cases. Our claim holds. ■